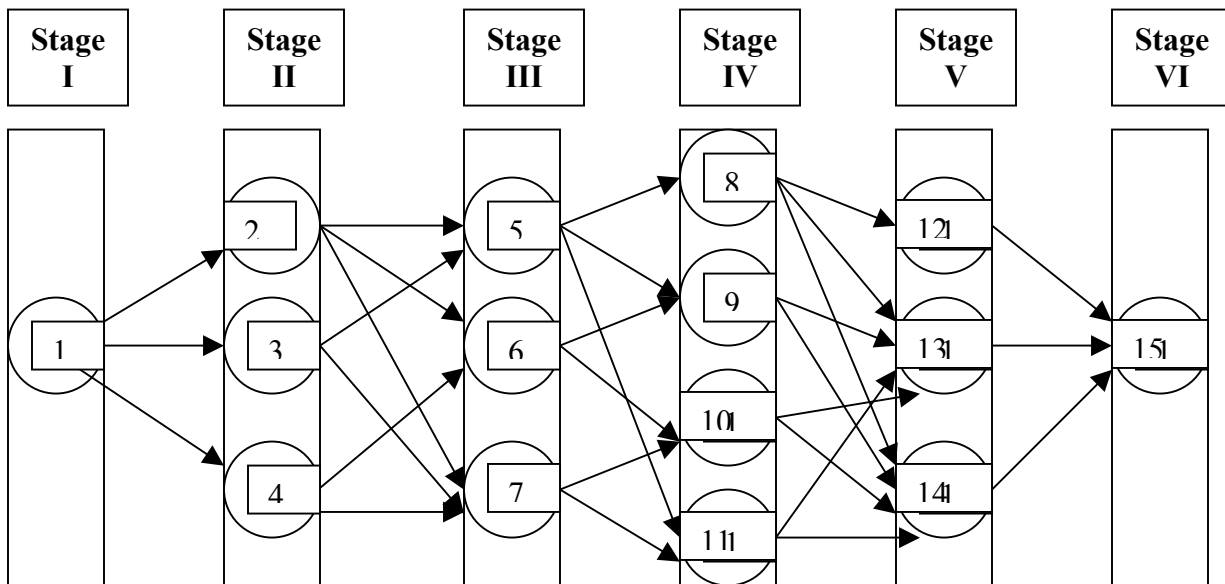


MULTISTAGE GRAPHS



The shortest cost journey from the source to a target in a graph with stages.

SAMPLE GRAPH AND SIMULATION OF THE ALGORITHM



THE COST MATRIX IS GIVEN BELOW

	I	II	III	IV	V	VI
	1	2	3	4	5	6
1		10	20	30		
2					10	20
3					40	50
4						40
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

SIMULATION OF THE SOLUTION USING BACKWARD COSTS

Format: COST(stage, node) = minimum cost of traveling to the node in stage from the source(node 1).

STEP I

$$\text{COST(I,1)} = 0$$

STEP II

$$\text{COST(II,2)} = \text{COST(I,1)} + \text{cost(1,2)} = 0 + 10 = 10$$

$$\text{COST(II,3)} = \text{COST(I,1)} + \text{cost(1,3)} = 0 + 20 = 20$$

$$\text{COST(II,4)} = \text{COST(I,1)} + \text{cost(1,4)} = 0 + 30 = 30$$

STEP III

$$\begin{aligned} \text{COST(III,5)} &= \min\{\text{COST(II,2)} + \text{cost(2,5)}, \\ &\quad \text{COST(II,3)} + \text{cost(3,5)}, \\ &\quad \text{COST(II,4)} + \text{cost(4,5)}\} \\ &= \min\{10 + 10, 20 + 40, 30 + \infty\} \\ &= 20 \text{---via the path 1-2-5} \end{aligned}$$

$$\begin{aligned} \text{COST(III,6)} &= \min\{\text{COST(II,2)} + \text{cost(2,6)}, \\ &\quad \text{COST(II,3)} + \text{cost(3,6)}, \\ &\quad \text{COST(II,4)} + \text{cost(4,6)}\} \end{aligned}$$

$$= \min\{10 + 20, 20 + \infty, 30 + 40\}$$

$$= 20 \text{---via the path 1-3-6}$$

$$\text{COST(III,7)} = \min\{\text{COST(II,2)} + \text{cost}(2,7),$$

$$\text{COST(II,3)} + \text{cost}(3,7),$$

$$\text{COST(II,4)} + \text{cost}(4,7)\}$$

$$= \min\{10 + 30, 20 + 50, 30 + 30\}$$

$$= 40 \text{---via the path 1-2-7}$$

STEP IV

$$\text{COST(IV,8)} = \min\{\text{COST(III,5)} + \text{cost}(5,8),$$

$$\text{COST(III,6)} + \text{cost}(6,8),$$

$$\text{COST(III,7)} + \text{cost}(7,8)\}$$

$$= \min\{20 + 10, 20 + \infty, 40 + \infty\}$$

$$= 30 \text{---via the path 1-2-5-8}$$

$$\text{COST(IV,9)} = \min\{\text{COST(III,5)} + \text{cost}(5,9),$$

$$\text{COST(III,6)} + \text{cost}(6,9),$$

$$\text{COST(III,7)} + \text{cost}(7,9)\}$$

$$= \min\{20 + 20, 20 + 20, 40 + \infty\}$$

$$= 40 \text{---via the path 1-2-5-9 or via the path 1-3-6-9}$$

$$\text{COST(IV,10)} = \min\{\text{COST(III,5)} + \text{cost}(5,10),$$

$$\text{COST(III,6)} + \text{cost}(6,10),$$

$$\text{COST(III,7)} + \text{cost}(7,10)\}$$

$$= \min\{20 + 10, 20 + \infty, 40 + \infty\}$$

$$= 30 \text{---via the path 1-2-5-10}$$

$$\text{COST(IV,11)} = \min\{\text{COST(III,5)} + \text{cost}(5,11),$$

$$\text{COST(III,6)} + \text{cost}(6,11),$$

$$\text{COST(III,7)} + \text{cost}(7,11)\}$$

$$= \min\{20 + 30, 20 + 30, 40 + 30\}$$

$$= 50 \text{---via the path 1-2-5-11 or via the path 1-3-6-11}$$

STEP V

$$\text{COST(V,12)} = \min\{\text{COST(IV,8)} + \text{cost}(8,12),$$

$$\text{COST(IV,9)} + \text{cost}(9,12),$$

$$\text{COST(IV,10)} + \text{cost}(10,12),$$

$$\text{COST(IV,11)} + \text{cost}(11,12)\}$$

$$= \min\{30 + 10, 40 + \infty, 30 + \infty, 50 + \infty\}$$

= 40—via the path 1-2-5-8-12

$$\begin{aligned} \text{COST}(V,13) &= \min\{\text{COST}(IV,8) + \text{cost}(8,13), \\ &\quad \text{COST}(IV,9) + \text{cost}(9,13), \\ &\quad \text{COST}(IV,10) + \text{cost}(10,13), \\ &\quad \text{COST}(IV,11) + \text{cost}(11,13)\} \\ &= \min\{30+20, 40+20, 30+10, 50+10\} \\ &= 40\text{---via the path 1-2-5-8-13} \end{aligned}$$

$$\begin{aligned} \text{COST}(V,14) &= \min\{\text{COST}(IV,8) + \text{cost}(8,14), \\ &\quad \text{COST}(IV,9) + \text{cost}(9,14), \\ &\quad \text{COST}(IV,10) + \text{cost}(10,14), \\ &\quad \text{COST}(IV,11) + \text{cost}(11,14)\} \\ &= \min\{30+30, 40+10, 30+20, 50+30\} \\ &= 50\text{---via the path 1-2-5-10-14 or via 1-2-5-9-14 or 1-3-6-9-14} \end{aligned}$$

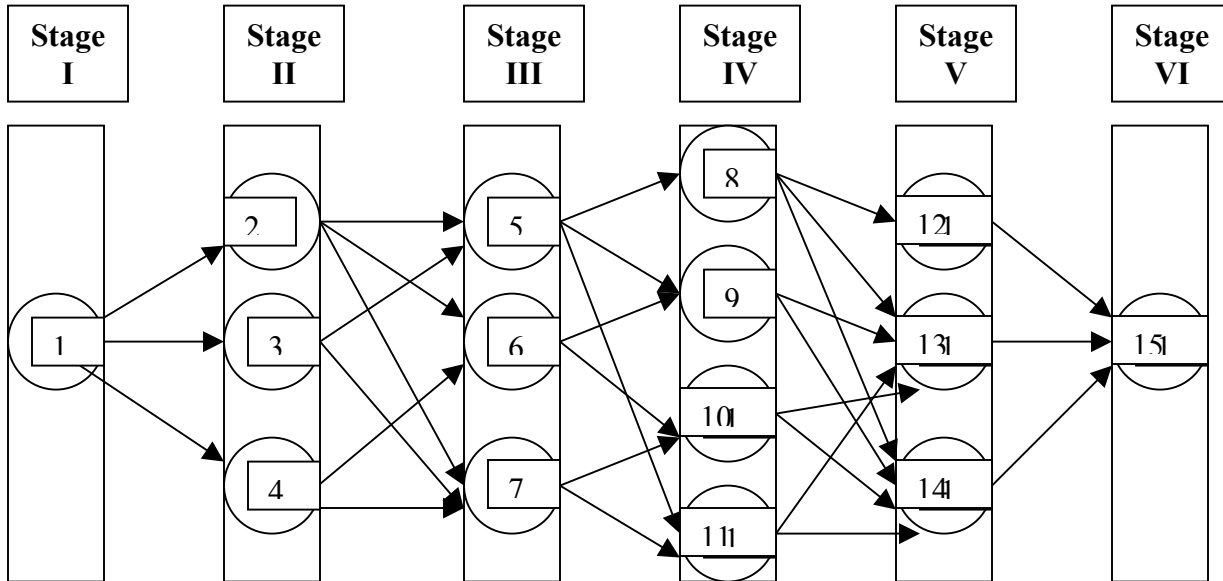
STEP VI

$$\begin{aligned} \text{COST}(VI,15) &= \min\{\text{COST}(V,12) + \text{cost}(12,15), \\ &\quad \text{COST}(V,13) + \text{cost}(13,15), \\ &\quad \text{COST}(V,14) + \text{cost}(14,15)\} \\ &= \min\{40+20, 40+10, 50+30\} \\ &= 50\text{---via the path 1-2-5-8-13-15} \end{aligned}$$

SIMULATION OF THE SOLUTION USING FORWARD COSTS

FORMAT: COST(stage, node)=minimum cost of travel from the node in stage to the target.

SAMPLE GRAPH AND SIMULATION OF THE ALGORITHM



THE COST MATRIX IS GIVEN BELOW

	I	II			III			IV				V		VI	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1		10	20	30											
2					10	20	30								
3					40		50								
4						40	30								
5								10	20	30	40				
6									20	30					
7										30	20				
8												10	20	30	
9													20	10	
10													10	20	
11													10	30	
12															20
13															10
14															30
15															

STEP I

COST(V,12)=20

COST(V,13)=10

COST(V,14)=30

STEP II

$$\begin{aligned} \text{COST(IV,8)} &= \min\{10 + \text{COST(V,12)}, 20 + \text{COST(V,13)}, 30 + \text{COST(V,14)}\} \\ &= \min\{10 + 20, 20 + 10, 30 + 30\} \\ &= 30\text{---via the path 8-12-15 or via the path 8-13-15} \end{aligned}$$

$$\begin{aligned} \text{COST(IV,9)} &= \min\{20 + \text{COST(V,13)}, 10 + \text{COST(V,14)}\} \\ &= \min\{20 + 10, 10 + 30\} \\ &= 30\text{---via the path 9-13-15} \end{aligned}$$

$$\begin{aligned} \text{COST(IV,10)} &= \min\{10 + \text{COST(V,13)}, 20 + \text{COST(V,14)}\} \\ &= \min\{10 + 10, 20 + 30\} \\ &= 20\text{---via the path 10-13-15} \end{aligned}$$

$$\begin{aligned} \text{COST(IV,11)} &= \min\{10 + \text{COST(V,13)}, 30 + \text{COST(V,14)}\} \\ &= \min\{10 + 10, 30 + 30\} \\ &= 20\text{---via the path 11-13-15} \end{aligned}$$

STEP III

$$\begin{aligned} \text{COST(III,5)} &= \min\{10 + \text{COST(IV,8)}, 20 + \text{COST(IV,9)}, 30 + \text{COST(IV,10)}, 40 + \text{COST(IV,11)}\} \end{aligned}$$

$$\begin{aligned} &= \min\{10 + 30, 20 + 30, 30 + 20, 40 + 20\} \\ &= 40\text{---via the path 5-8-12-15 or 5-8-13-15} \end{aligned}$$

$$\begin{aligned} \text{COST(III,6)} &= \min\{20 + \text{COST(IV,9)}, 30 + \text{COST(IV,10)}\} \\ &= \min\{20 + 30, 30 + 20\} \\ &= 50\text{---via the path 6-9-13-15 or 6-10-13-15} \end{aligned}$$

$$\begin{aligned} \text{COST(III,7)} &= \min\{30 + \text{COST(IV,10)}, 20 + \text{COST(IV,11)}\} \\ &= \min\{30 + 20, 20 + 20\} \\ &= 40\text{---via the path 7-11-13-15} \end{aligned}$$

STEP IV

$$\begin{aligned} \text{COST(II,2)} &= \min\{10 + \text{COST(III,5)}, 20 + \text{COST(III,6)}, 30 + \text{COST(III,7)}\} \\ &= \min\{10 + 40, 20 + 50, 30 + 40\} \\ &= 50\text{---via the path 2-5-8-12-15 or 2-5-8-13-15} \end{aligned}$$

$$\begin{aligned} \text{COST(II,3)} &= \min\{40 + \text{COST(III,5)}, 50 + \text{COST(III,7)}\} \\ &= \min\{40 + 40, 50 + 40\} \\ &= 80\text{---via the path 3-5-8-12-15 or 3-5-8-13-15} \end{aligned}$$

$$\begin{aligned} \text{COST(II,4)} &= \min\{40 + \text{COST(III,6)}, 30 + \text{COST(III,7)}\} \\ &= \min\{40 + 50, 30 + 40\} \\ &= 70\text{---via the path 4-7-11-13-15} \end{aligned}$$

STEP V

$$\begin{aligned} \text{COST(I,1)} &= \min\{10 + \text{COST(II,2)}, 20 + \text{COST(II,3)}, 30 + \text{COST(II,4)}\} \\ &= \min\{10 + 50, 20 + 80, 30 + 70\} \\ &= 50\text{---via the path 1-2-5-8-12-15 or via the path 1-2-5-8-13-15} \end{aligned}$$