

ANALYSIS OF ALGORITHMS

ORDER NOTATION



Let $T(n) = 45n^4 - 5n^2 + 4n - 456$. Show $T(n) = O(T_1(n)) = \theta(T_1(n)) = \Omega(T_1(n))$ where $T_1(n) = n^4$. Obtain $G_1(n)$ and $G_2(n)$ where $T(n) = o(G_1(n)) = \omega(G_2(n))$.

**UPPER BOUND
BIG OH NOTATION
 $T(n) = O(U(n))$ if after $n = k$
 $T(n) < U(n)$**

DEFINITION:

$T(n) = O(U(n))$ iff there exists a constant k_u such that $U(n) > k_u T(n)$ for $n \geq n_1$

Now $T(n) = 45n^4 - 5n^2 + 4n - 456$

$< 45n^4 + 4n$, by throwing away the negative terms

$< 45n^4 + 4n^4$ by increasing the exponent of n

$< 49n^4$

Hence $T(n) = O(n^4)$ ---- UPPER BOUND

**LOWER BOUND
BIG OMEGA NOTATION
 $T(n) = \Omega(L(n))$ if after $n = k$
 $T(n) < L(n)$**

DEFINITION

$T(n) = \Omega(L(n))$ ---LOWER BOUND iff there exists a constant k_1 such that $T(n) > k_1 \Omega(L(n))$ for $n > n_1$

$$T(n) = 45n^4 - 5n^2 + 4n - 456$$

$$> 45n^4 - 5n^2 - 456 \text{ --- by throwing away the positive terms}$$

$$> 45n^4 (1 - (5n^2 - 456)/45n^4)$$

$$> 45n^4 (1 - 1/9n^2 - 456/45n^4)$$

$$> 45n^4 (1 - 1/9n^2 - 1/45n^4) \text{ ---- for } n > 456$$

$$> 45n^4 (1 - 1/9n^2 - 1/45n^2) \text{ ---- for } n > 456$$

$$> 45n^4/2$$

$$= \Omega(n^4) \text{-----LOWER BOUND}$$

THETA NOTATION
 $T(n) = \theta(S(n))$ if after $n = k$
 $S(n) < T(n) < U(n)$

DEFINITION

$T(n) = \theta(G(n))$ iff $G(n)$ is both an upper and lower bound.

Hence in the above case $T(n) = \theta(n^4)$

SMALL OH NOTATION

$$T(n) = O(G(n))$$

$$\lim_{n \rightarrow \infty} [T(n)/G(n)] = 0$$

DEFINITION

$T(n) = o(G(n))$ iff $\lim_{n \rightarrow \infty} T(n)/G(n) = 0$, here $G(n)$ has a higher functional growth,

Consider $G(n) = o(n^6)$. $\lim_{n \rightarrow \infty} [n^6/n^2] = 0$.

<p style="text-align: center;">SMALL OMEGA NOTATION</p> <p style="text-align: center;">$T(n) = \omega(G(n))$</p> <p style="text-align: center;">$\lim_{n \rightarrow \infty} [G(n)/T(n)] = 0$</p>
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DEFINITION:

$T(n) = \omega(G(n))$ iff $\lim_{n \rightarrow \infty} (G(n)/T(n)) = 0$ i.e. $G(n)$ has a lower functional growth than $T(n)$, here let $T(n) = n^3$. $\lim_{n \rightarrow \infty} [n^3/n^4] = 0$.