ANALYSIS OF ALGORITHMS

ORDER NOTATION

Let $T(n) = 45n^4-5n^2+4n-456$. Show $T(n)= O(T1(n))=\theta(T1(n))=\Omega(T1(n))$ where $T1(n)=n^{4.}$ Obtain G1(n) and G2(n) where $T(n)=o(G1(n))=\omega(G2(n))$.

UPPER BOUND
BIG OH NOTATION
T(n) = O(U(n)) if after n = k
T(n) < U(n)

DEFINITION:

$T(n) = O(U(n))$ iff there exists a constant k_u such that $U(n) > k_uT(n)$ for $n >= n1$

Now $T(n) = 45n^4-5n^2+4n-456$

$< 45n^4+4n$, by throwing away the negative terms

$< 45n^4+4n^4$ by increasing the exponent of n

$< 49n^4$

Hence $(n) =O(n^4)$ ---- UPPER BOUND

LOWER BOUND
BIG OMEGA NOTATION
T(n) = O(L(n)) if after n = k
T(n)<L(n)

DEFINITION

$T(n) = \Omega(L(n))$---LOWER BOUND iff there exists a constant k_l such that $T(n) >$ $k_l\, \Omega(L(n))$ for $n > n1$

$T(n) = 45n^4 - 5n^2 + 4n - 456$

$> 45n^4 - 5n^2 - 456$ --- by throwing away the positive terms

$> 45n^4(1-(5n^2-456)/45n^4)$

$> 45n^4\ (1 - 1/9n^2\ -456/45n^4)$

$> 45n^4\ (1 - 1/9n^2\ -1/45n^4)$ ---- for $n > 456$

$> 45n^4\ (1 - 1/9n^2\ -1/45n^2)$ ---- for $n > 456$

$> 45n^4/2$

$= \Omega(n^4)$------LOWER BOUND

THETA NOTATION

$T(n) = \theta(S(n))$ if after $n = k$

$S(n) < T(n) < U(n)$

DEFINITION

$T(n) = \theta(G(n))$ iff $G(n)$ is both an upper and lower bound.

Hence in the above case $T(n) = \theta(n^4)$

SMALL OH NOTATION

$T(n) = O(G(n))$

$\text{Lim}_{n \to \infty}[T(n)/G(n)] = 0$

DEFINITION

$T(n) = o(G(n))$ iff $Lim_{n\to\infty(}T(n)/G(n)) =0$, here G(n) has a higher functional growth,

Consider $G(n) = o(n^6)$. $Lim_{n \to \infty} [n^6/n^2] = 0$.

SMALL OMEGA NOTATION

$T(n) = \omega(G(n))$

$Lim_{n \to \infty}[G(n)/T(n)] = 0$

DEFINITITON:

$T(n) = \omega(G(n))$ iff $Lim_{n \to \infty} ((G(n)/T(n)) = 0$ i.e. G(n) has a lower functional growth than T(n), here let $T(n) = n^{3.}$ $Lim_{n \to \infty} [n^3/n^4] = 0$.