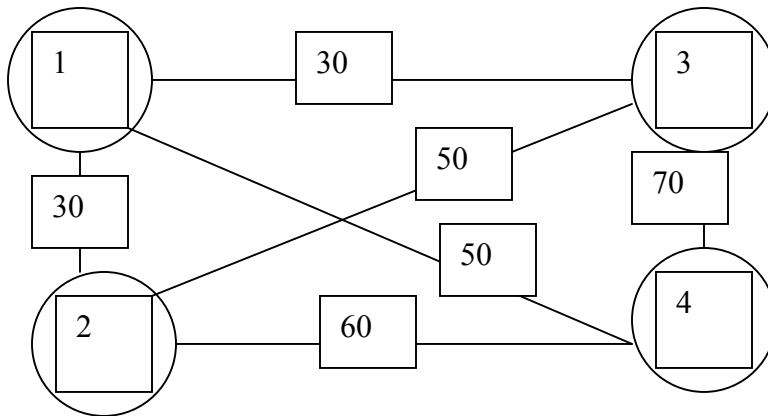


# TRAVELLING SALESMAN PROBLEM DYNAMIC PROGRAMMING SOLUTION



## SAMPLE GRAPH

$$\text{COST}(k,l) = 10(k+l)$$



### THE TRAVELLING SALESMAN PROBLEM

In a complete weighted graph find a tour of minimum cost.

Let a journey be defined as follows. A journey  $J(k,S)$  starts at a node  $k$ , passes through all the nodes in  $S$  at most once and terminates at node 1.  $S$  is a subset of the nodes of the graph excluding node 1.

$S$  is the null set

$J(2,\varnothing) = 30, J(3,\varnothing) = 40, J(4,\varnothing) = 50$ ---direct paths to node 1.

$S$  has one element

$$J(2, \{3\}) = \text{cost}(2,3) + J(3, \varnothing) = 50 + 30 = 80$$

$$J(2, \{4\}) = \text{cost}(2,4) + J(4, \varnothing) = 60 + 40 = 100$$

$$J(3, \{2\}) = \text{cost}(3,2) + J(3, \varnothing) = 50 + 30 = 80$$

$$J(3, \{4\}) = \text{cost}(3,4) + J(4, \varnothing) = 70 + 40 = 110$$

$$J(4, \{2\}) = \text{cost}(4,2) + J(2, \varnothing) = 60 + 30 = 90$$

$$J(4, \{3\}) = \text{cost}(4,3) + J(3, \varnothing) = 70 + 30 = 100$$

S has two elements

$$J(2, \{3,4\}) = \min\{\text{cost}(2,3) + J(3, \{4\}), \text{cost}(2,4) + J(4, \{3\})\} = \min(50 + 110, 60 + 100) \\ = \min(160, 160) = 160$$

$$J(3, \{2,4\}) = \min\{\text{cost}(3,2) + J(2, \{4\}), \text{cost}(3,4) + J(4, \{3\})\} = \min(50 + 100, 70 + 100) \\ = \min(150, 170) = 150$$

$$J(4, \{2,3\}) = \min\{\text{cost}(4,2) + J(2, \{3\}), \text{cost}(4,3) + J(3, \{2\})\} = \min(60 + 80, 70 + 80) \\ = \min(140, 150) = 140$$

S has three elements

$$J(1, \{2,3,4\}) = \min\{\text{cost}(1,2) + J(2, \{3,4\}), \text{cost}(1,3) + J(3, \{2,4\}), \\ \text{cost}(1,4) + J(4, \{2,3\})\} \\ = \min\{30 + 160, 40 + 150, 50 + 140\} \\ = \min(190, 190, 190) \\ = 190$$

**THE MINIMUM COST TOURS ARE THUS**

**1-2-4-3-1 OR 1-3-2-4-1 OR 1-4-2-3-1  
OR 1-3-4-2-1**

## SAMPLE GRAPH

$$\text{COST}(k,l) = 10(k+l)$$

